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Numerical Analysis of Mechanical Engineering Nonlinear Normal Modes

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Abstract

In this study, we take a look back at how NNMs have progressed in terms of computational approaches in the recent past. Both undamped and damped NNM calculation techniques are shown, and their benefits and drawbacks are explored. Applications ranging from low-dimensional weakly nonlinear systems to high-dimensional substantially nonlinear industrial structures are used to demonstrate the approaches. Nonlinear normal modes, periodic orbits, invariant manifolds, and numerical continuation are some of the key concepts that will be covered in this article.

1 Introduction

Modal analysis is used to calculate or measure a structure's inherent frequencies and vibration modes [1]. Commonly used in the manufacturing sector for both design and certification purposes. For airplanes, verifying the accuracy of the model for flutter calculations requires comparing the observed modal parameters from ground vibration testing with those predicted by the numerical model [2]. Structural nonlinearities are often found during test campaigns (e.g., for the Airbus A400M [3] and for the F-16 aircraft [4]), while being frequently neglected or missed during the design

stage. New dynamical occurrences that have no linear analogue may be discovered, which is one of the main difficulties posed by nonlinearity. Nonlinearity, on the other hand, is said to open up new possibilities for improving system performance, according to a recent body of research. Acoustic switches and rectifiers [5, 6], micromechanical oscillators [7, 8], granular media [7, 9], and vibration absorbers [10, 11] all make use of nonlinearity on purpose.

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There is an obvious need here for generalizing the framework of linear modal analysis to nonlinear systems. Nonlinear normal modes (NNMs) cannot be superposed like linear modes since they are not orthogonal to one another and do not decouple the equations of motion. Therefore, a practitioner may doubt their usefulness. Mode bifurcations [11] and modal interactions [10] between widely dispersed modes cannot be described by linear theory, but they are still crucial dynamical occurrences. Even state-of-the-art linear experimental modal analysis techniques have trouble with nonlinearity [12], sometimes identifying two linearized modes when only one nonlinear mode is activated. Therefore, we consider the NNM theory to be a useful tool for structural dynamicists. The frequency-energy dependence, interactions, and bifurcations of NNMs were previously unknown, but were uncovered by the first analytical efforts to calculate NNMs [11, 13–16]. However, high-dimensional structures are difficult to analyze using analytical techniques. The past ten years have seen the rise of NNM-specific numerical techniques, thanks to both increases in processing power and in computer approaches (especially in numerical continuation [17–21]). However, there are now algorithms that can be applied to genuine engineering structures, giving the type of in-depth analysis necessary for a thorough treatment of nonlinear systems, suggesting that this field of study is maturing. Recent literature [22–24] has reviewed the theory and

applications of NNMs, but this study is the first to conduct a comprehensive and critical evaluation of the many families of computational techniques for both undamped and damped NNMs.

2 The paper will proceed as described below. In Section 2, we take a quick look back at the two primary definitions of NNMs and their frequency-energy dependency. In Section 3, we go through several numerical techniques for calculating the undamped NNMs (periodic movements) of Hamiltonian systems. The calculation of damped NNMs, which are phase-space 2-manifolds with damping, is discussed in Section 4. At the end of each chapter, a detailed analysis of the strategies discussed is provided. Section 5 concludes the research and discusses the outstanding issues and difficulties that will hopefully propel the growth of nonlinear modal analysis in the future.

3 There is a large body of work on the topic of numerical computing of periodic solutions and manifolds (e.g., [25–33]). Many of the techniques now in use for NNM calculation were taken directly from this body of work. Examples include the trajectories-based technique mentioned in Section 4.2.1 and



the firing and pseudo-arclength continuation discussed in Section 3. The emphasis of this study is on nanoscale materials and their applications in mechanical engineering. For this reason, no effort has been made to provide

a comprehensive history of these techniques and their many applications, such as in computing forced responses and invariant manifolds of broad nonlinear systems

4. Non-Negative-Number-Matters (NNMs): Definition and Frequency-Energy

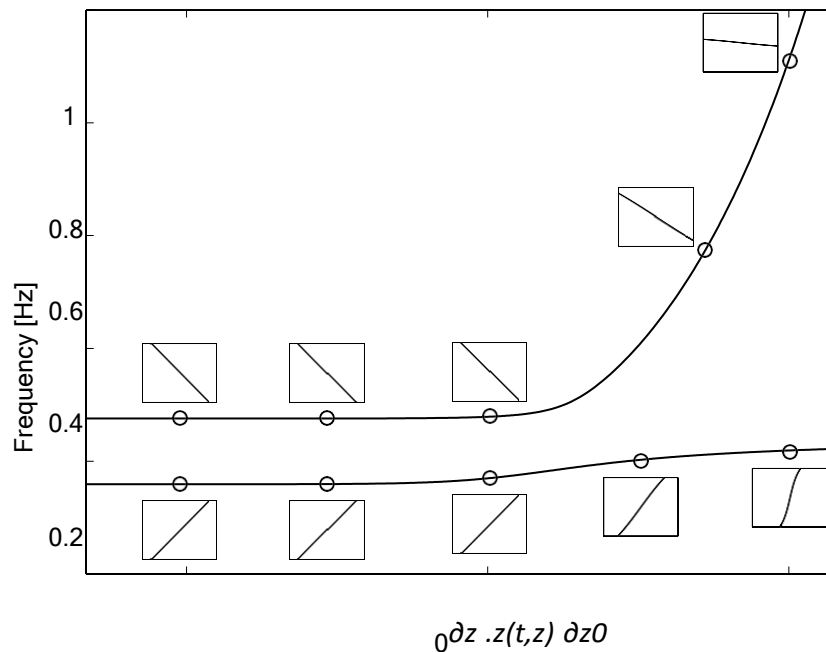
Lyapunov and Poincar'e created the groundwork that was crucial to the development of the nonlinear normal mode (NNM) theory. According to Lyapunov's center theorem, a finite-dimensional Hamiltonian system has multiple families of periodic solutions near the stable equilibrium point of the system, which can be understood as nonlinear extensions of the normal modes of the underlying linear system, provided certain regularity assumptions and non-resonance conditions are met [34, 35]. Rosenberg described an undamped NNM in this theoretical context as a system-wide vibration (i.e., a synchronous periodic oscillation). According to this concept, it is necessary for all significant system points to simultaneously attain their extreme values and pass through zero. All dislocations may be written in terms of a single standard dislocation. In addition, Rosenberg was the pioneer in suggesting creative methods for determining NNMs [36–38]. However, modal interactions, in which the periodic motion includes the frequencies of at least two interacting modes, cannot be captured by this description. This review study focuses on the definition of an undamped NNM that was presented in [39], which expands the concept of a NNM to include (non-necessarily synchronous) periodic movements.

In light of the fact that the frequencies of nonlinear oscillations rely on the energy of the system, the frequencies, modal shapes, and periodic movements of NNMs are also energy dependent. The in-phase and out-of-

phase NNMs of a two-degree-of-freedom (DOF) system are shown in Figure 1(a), which depicts the backbone curve of the NNMs.

For expanding energies, $x_1 + (2x_1 x_2) + 0.5 x_3 = 0$ $x_2 + (2x_2 x_1) = 0$ (1). Similarities exist between the low-energy NNM movements and the corresponding mode of the underlying linear system. In phase space, the periodic motion takes the form of an ellipse, whereas the modal shape is a straight line. At higher energies, the modal shape curves, and the corresponding phase-space motion takes on an elliptical distortion. The period of the motion shortens as the energy increases because of the hardening feature of the cubic spring.

Additional branches of periodic solutions emerge from the backbone curve if the energy is raised for the in-phase NNM, as seen in Figure 1(b). These interactions are a concrete example of $n:m$ couplings between in-phase and quadrature modes. Figure 2 illustrates the frequency ratio of the out-of-phase NNMs to the in-phase NNMs, which rises from 3 to $+$, providing an explanation for the emergence of these modal interactions. When the ratio is three to one, for instance, the third harmonics of the in-phase mode have the same frequency as the fundamental harmonics of the anti-phase mode, fulfilling the criteria for a 3:1 resonance. Figure 2 shows that system (1) has an infinite number of possible paths for modal interactions [22]. Because of their importance to engineering, NNM's overarching goal is to



3.1.1 at $t = T$. Therefore, it may be calculated by numerically integrating (11) from zero-based starting points. Large-scale models greatly benefit from the reduced processing cost as compared to finite differences. Nonlinear equations of motion (3) are solved with equations (11) in a single numerical simulation in Ref. [44]. This significantly reduces the computational cost by requiring just one extra iteration each time step for the calculation of the monodromy matrix.

3.1.2 The stability of the NNM movements is determined by the Floquet multipliers, which are the

eigenvalues of the monodromy matrix. If the amplitude of the floquet multiplier is larger than 1, the motion is unstable.

3.1.3 Discretization Methods

The challenge of finding a periodic orbit is transformed into solving a set of nonlinear algebraic equations using discretization procedures, in contrast to shooting methods that depend on temporal integration to estimate a periodic solution using trial (or, equivalently, shape) functions. In particular, the series approximating the trajectory $z(t)$ on the interval $[0, T]$ Nonlinear force $f_{nl}(x)$ is approximated by the Fourier series $\sum_{n=0}^{\infty} b_n(z, \omega)$. To determine whether nonlinearities are nontrivial or not differentiable, the HB technique uses a Fourier expansion of the



nonlinear force vector $\text{fnl}(x)$, which may be time-consuming or perhaps impossible for large-scale problems. Many tweaks to the original algorithm were offered to fix this issue. For instance, the linearized issues that arise during the continuation phase are the only ones for which the harmonics are balanced by the incremental HB approach [49]. One alternative is to use the Fast Fourier Transform and its inverse to hop back and forth between the time and frequency domains. Due to the time-domain computation and frequency-domain transformation of the force vector $\text{fnl}(x)$, this approach may be used to handle any nonlinearity [50]. System (3) is recast into quadratic form using a high-order frequency-based HB approach, which was suggested by Cochelin and Vergez in Ref. [51].

(21) $d(z, t) = c + l_1(z) + l_2(z, z)$, with the linear operators d and l_1 and the bilinear operator l_2 all being present. In order to do this, it is common to need to collect extra (artificial) variables alongside the state space variables z in the form of an extended vector z . The quadratic recast makes it possible to obtain the truncated Fourier series expansion of $\text{fnl}(x)$ for a wide variety of nonlinear systems in a formal and systematic way using just elementary algebra.

The n_H number of harmonics used in the Fourier expansion is a key technique parameter. This value may be known in front of the calculation, or it may change while the branch of periodic orbits progresses. In contrast to local techniques, which take into account a different number of harmonics for each degree of freedom (DOF), global methods impose the same number of harmonics across all DOFs. Since not all degrees of freedom (DOFs) are subjected to the same amount of nonlinear distortions, the later approaches are especially useful. For further information on these methods, see Ref. [52]. The monodromy matrix is not a result of the HB technique, in contrast to shooting, which is intrinsically a time-domain approach.

But Hill's technique [53, 54] allows for direct analysis of stability in the frequency domain. To find out whether a periodic solution is stable, we solve an eigenvalue problem whose answers provide us direct access to the so-called Floquet exponents (connected to the Floquet multipliers).

Orthogonal Collocation

Orthogonal collocation, in contrast to the HB technique, is a time domain approach that uses order- p Lagrange polynomials as trial functions and divides the time interval into n_t smaller intervals $t_j = [t_j, t_{j+1}]$ (called mesh intervals). For each interval of the mesh, the solutions to equation (3) are colocated at Gauss points (that is, using a Dirac distribution for the test function $i(t)$). By combining the periodicity constraint with the discretized equations of motion, we have an algebraic system with $2N$ ($p_{nt} + 1$) equations, which can be solved using a Newton-like method. This system's size is Since the values of $z(t)$ at all collocation sites are unknown, this problem is substantially more extensive than the shooting approach (6). On the other side, there is no need for direct numerical integration. Interestingly, the elements of the matrix that stands in for the discretized issue may be used to calculate the monodromy matrix. Its eigenvalues may be used for stability analysis of a periodic solution, much as the shooting technique.

3.2 Continuation of Periodic Solutions

Nonlinear equations of the type $H(q, \lambda) = 0$ where H is a vector of n_q equations, q is a vector of n_q unknowns (such as $z(0)$ or z), and λ is a single control parameter are used in the numerical techniques provided in Section 3.1 to establish a periodic solution. The system of equations that needs solving includes the term $H(U) = 0$ (22), where U is the extended vector defined as $[q, \lambda]$ to the

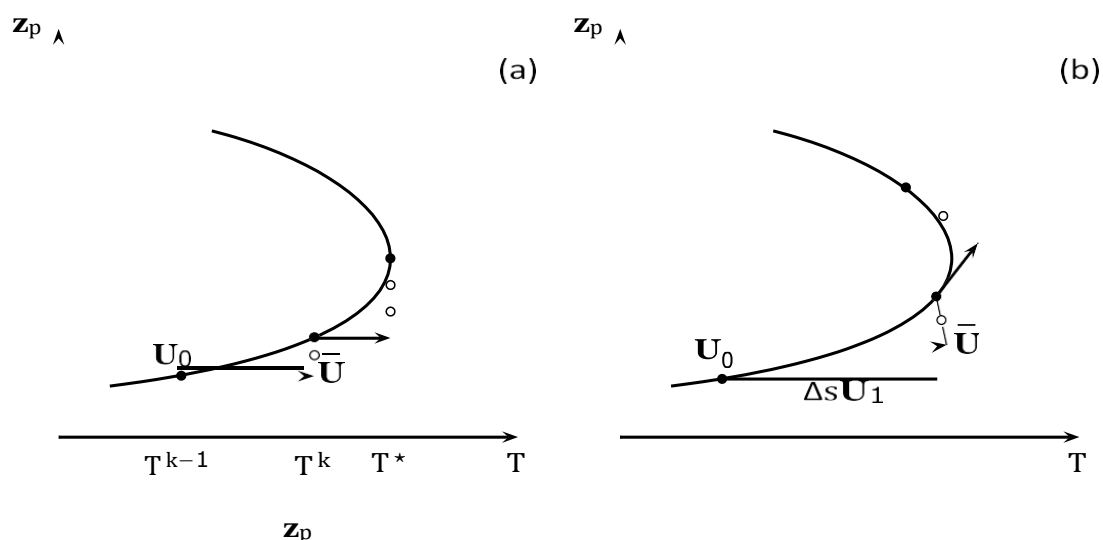


power of U $Rnq+1$. Provided that the solution to system (22) is not a bifurcation point, the presence of a locally unique continuum of periodic solutions, known as a solution branch (or solution family) [18], is guaranteed by the implicit function theorem.

These branches of the answers may be found from any given starting point using the continuation (or path following) method. A first prediction is obtained by (i) a predictor step, which infer U to the answer of (22), and (ii) an iterative corrector that refines this first approximation to find an exact answer to Eqs. (22). The order of approximation utilized in the forecast and the limits put on the correction vary among the several existing continuation techniques. Many approaches exist (for example, see Ref. [19]), however for NNM calculations, only three have been widely used: sequential, pseudo-arclength, and asymptotic numeric continuation. The nonlinear system of equations (5) describing the periodic solutions for conservative autonomous

systems has the same number of equations as unknowns and no free control parameter for the continuation. The method presented in Ref. [29] involves the introduction of an arbitrary damping parameter v and the subsequent search for periodic solutions of the dissipative system in order to recast the system under the form (22).

$z' = g(z) + v\nabla s(z)$. (23) where z represents the whole system's energy. Only if $v = 0$ do periodic solutions of (23) exist, as shown in [29]. This means that throughout the continuing process, the parameter will always seem to be equal to zero. However, the addition of this arbitrary component does have the desirable side effect of recasting the continuation problem as a square with the same number of equations and variables. The calculation of NNMs was simplified in Ref. [45] by using linear viscous damping instead. The Moore-Penrose pseudo-inverse, as in Peeters et al. [44], is another option for solving the overdetermined system of equations.



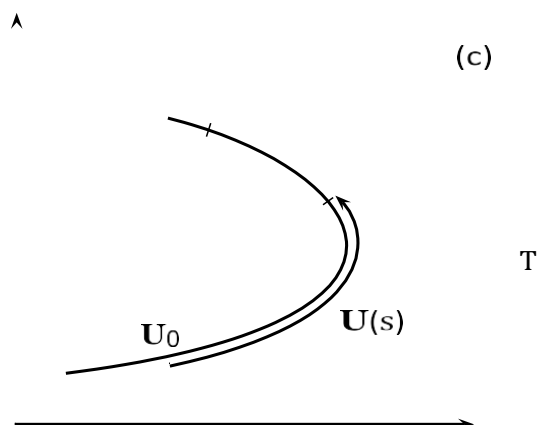


Figure 4: Schematic procedure of the predictor (→) and corrector (---) steps followed in the (a) sequential; (b) pseudo-arclength; (c) asymptotic-numeric continuation methods. (---) Branch including a turning point.

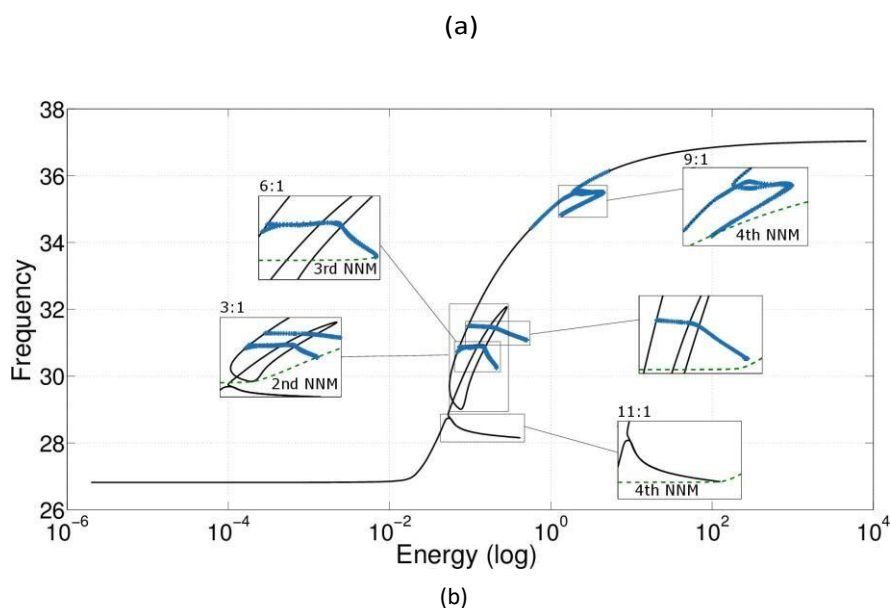


Figure : Clamped-simply-supported beam with a bilateral contactor. (a) Schematic of the system; (b) FEP of the first NNM. Figure reproduced from [87]. distinct forecasts, as the Moore-Penrose technique used in MATCONT [91]. In [95], the authors examine three different methods for correcting the NNMs of a 2DOF system and present their findings. Both structural dynamics [96] and fluid dynamics [97, 98] made use of the more modern ANM continuation method. Using MANLAB's HB technique and ANM, we were able to

acquire the findings shown in the figures. furthermore, there is no need for a sequential control procedure while using this technique. In addition, if the original system is recast into quadratic form (as in Eqs. (21), the cost of this high-order technique remains minimal. However, automated differentiation [100] maybe used in lieu of this a priori laborious process in order to automatically generate the Taylor series. Most enticingly, the bifurcations along the branch of solutions may be detected and characterized by analyzing the



ensuing Taylor series. This ensures the continuation algorithm's stability even when subjected to a large number of internal resonances. Although this presentation focuses on ways to compute NNMs of conservative autonomous systems, it is evident that this is not a complete list. For example,

in Ref. [101] two techniques are offered that are quite similar to the procedures detailed in Chapter 4. The first strategy uses beginning circumstances with small amplitudes.

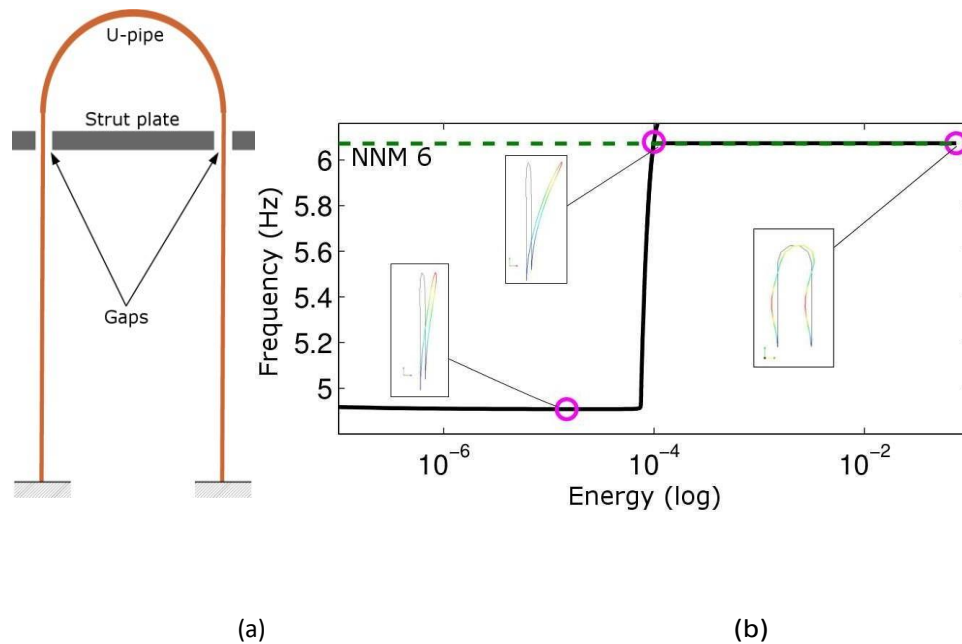
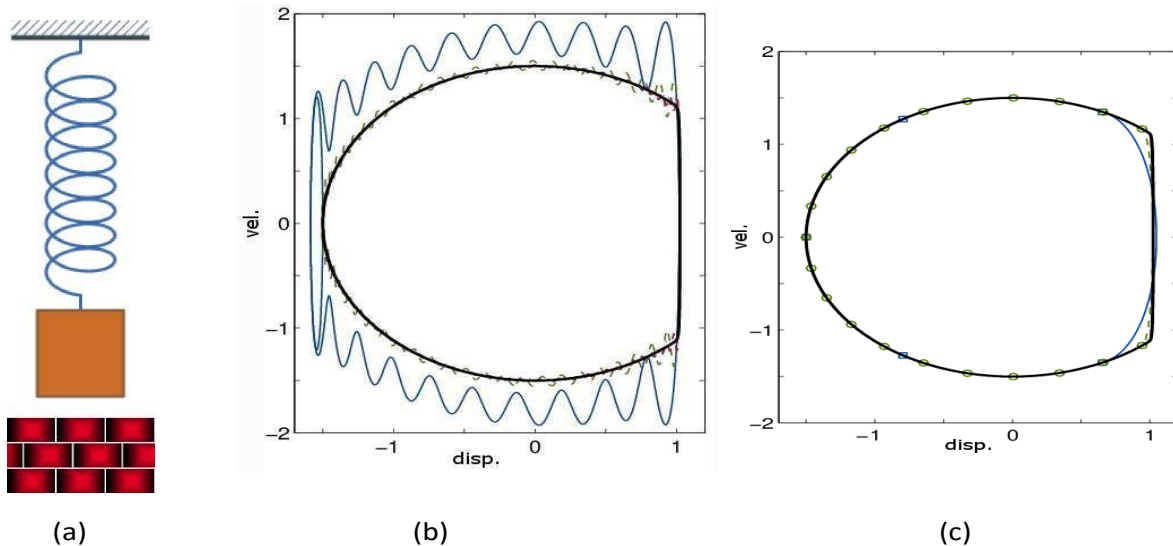


Figure 10: Simplified model of a vertical U-pipe of a steam generator. (a) Schematic of the system; (b) FEP of the first out-of-plane bending mode. Figure reproduced from [87].



CONCLUSION

There are several unanswered concerns about the interpretation and use of the calculated periodic orbits and invariant manifolds, in addition to the computational aspects: Damping's function in the dynamics of nonlinear systems is still poorly understood. The subject of how damping impacts modal interactions and invariant manifolds, for example, is still largely unsolved. Nonlinear systems rely heavily on invariant objects, which operate as dynamical attractors. Invariant manifold computations have been performed, but the physical knowledge they provide has not been completely used. Recent efforts have been made to conduct continuation in physical experiments [141-143] and to discover NNMs from experimental data [139, 140]. Validation procedures for nonlinear structural models might be greatly improved by combining the aforementioned improvements with the already-existing techniques for NNM calculation.

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